## Exercises for Chapter 2

2.1 Bennett and Franklin [1954] present the following data where the 10 determinations of the percentage of chlorine were made in two different batches of a polymer.

| Determination |  | Batch 1 |  | Batch 2 |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  | 58.59 |  | 55.71 |
| 2 |  | 58.45 |  | 56.65 |
| 3 |  | 59.64 |  | 56.72 |
| 4 |  | 58.64 |  | 57.56 |
| 5 |  | 58.00 |  | 58.27 |
| 6 |  | 57.03 |  | 56.58 |
| 7 |  | 57.33 |  | 57.08 |
| 8 |  | 57.80 |  | 57.13 |
| 9 |  | 58.04 |  | 57.92 |
| 10 |  | 58.41 |  | 56.21 |

(a) Compute the sample mean and sample variance of the ten determinations for each batch.
(b) Make a normal probability plot of the determinations for each batch separately, and comment on whether the data appears to follow a normal distribution.
(c) Calculate the pooled standard deviation, sp .
(d) Calculate the two-sample $t$-statistic for testing whether the percentage of chlorine is the same in the two batches of polymer.
(e) Calculate the critical $\mathrm{t}_{18,0.05}$
(f) Is there a significant difference between the two batches with respect to the percentage of chlorine?
2.2 Guttman, et. al. [1965] present the data below of the determinations of the ratio of iodine to silver in four different silver preparations.

| Preparation A | Preparation B | Preparation C | Preparation D |
| :---: | :---: | :---: | :---: |
| 1.176422 | 1.176411 | 1.176429 | 1.176449 |
| 1.176425 | 1.176441 | 1.176420 | 1.176450 |
|  |  | 1.176437 |  |

(a) Calculate the ANOVA table to compare the iodine to silver ratios between the four silver preparations.
(b) How is the F-statistic calculated?
(c) What is the critical value of the F-distribution to which the calculated statistic should be compared?
(d) Is there a significant difference in the four preparations?
2.3 Construct a frequency distribution (histogram) of the bore diameter data given below.

| 0.5755 | 0.5750 | 0.5748 | 0.5755 |
| :--- | :--- | :--- | :--- |
| 0.5753 | 0.5753 | 0.5754 | 0.5751 |
| 0.5756 | 0.5758 | 0.5758 | 0.5754 |
| 0.5762 | 0.5760 | 0.5757 | 0.5755 |
| 0.5756 | 0.5756 | 0.5759 | 0.5756 |
| 0.5758 | 0.5755 | 0.5762 | 0.5760 |
| 0.5758 | 0.5759 | 0.5760 | 0.5754 |
| 0.5768 | 0.5755 | 0.5761 | 0.5755 |
| 0.5750 | 0.5755 | 0.5750 | 0.5755 |
| 0.5753 | 0.5752 | 0.5752 | 0.5750 |
| 0.5750 | 0.5753 | 0.5750 | 0.5751 |
| 0.5757 | 0.5759 | 0.5759 | 0.5756 |
| 0.5759 | 0.5755 | 0.5759 | 0.5754 |
| 0.5753 | 0.5756 | 0.5756 | 0.5756 |
| 0.5759 | 0.5758 | 0.5760 | 0.5759 |
| 0.5761 | 0.5760 | 0.5758 | 0.5753 |
| 0.5750 | 0.5765 | 0.5761 | 0.5765 |
| 0.5763 | 0.5755 | 0.5760 | 0.5755 |
| 0.5757 | 0.5761 | 0.5769 | 0.5768 |

2.4 A Simulation program for mating tolerances calls a subroutine that generates a random number that is equally likely to be any value between 2.49 and 2.51 .
a) What is the probability that the subroutine generates a number between 2.492 and 2.498?
b) What is the probability that the subroutine generates a number greater than 2.504 ?
2.5 Contact window forming is one of the more critical steps in fabricating state of the art CMOS integrated circuits. The purpose of the windows is to facilitate connections between the gates sources and drains. The diameter length of the window is a critical dimension which is produced by a photo lithography and etching processes. The distribution of the diameters is Normal with mean $\mu=3.5: \mathrm{m}$ and standard deviation $\sigma=.15: \mathrm{m}$.
a) Find the probability that a contact window diameter is between $3.3 \mu \mathrm{~m}$ and $3.6 \mu \mathrm{~m}$.
b) Find the probability that a contact window diameter is between $3.1 \mu \mathrm{~m}$ and $3.4 \mu \mathrm{~m}$.
c) Find the probability that a contact window diameter is greater than $3.6 \mu \mathrm{~m}$.
2.6 You have a continuous random variable Y with density function:

$$
f(Y)=0.5 Y \text { for } 0 \leq Y \leq 2, \quad \text { and } f(Y)=0 \text { otherwise }
$$

a) Sketch the graph of the density function.
b) Find $\mathrm{P}\{\mathrm{Y} \geq 1\}$
c) Find the mean or $\mu=\mathrm{E}(\mathrm{Y})$ and indicate where it is on the graph you made in a)
d) Compute the variance of Y.
2.7 On a graph mark a horizontal scale from -10 to 10 and on this scale sketch the density function of the following distributions
a) Normal with Mean $\mu=0$, and standard deviation $\sigma=1$.
b) Normal with mean $\mu=0$, and standard deviation $\sigma=2$
c) Normal with mean $\mu=2$, and standard deviation $\sigma=1$.

What does this exercise tell you about the mean and standard deviation in relation to the shape of a Normal Distribution?
2.8 The data below came from a voltage-endurance test of an insulating fluid, and represents time to breakdown at 35 kV .
$16,33,41,87,93,98,134,258$
a) Calculate the median, $25^{\text {th }}$ percentile and $75^{\text {th }}$ percentile of the data
b) Construct a boxplot of the data.
c) Calculate the sample mean $\bar{Y}$, of the data.
d) Calculate the variance and the standard deviation.
e) Make a dot plot of the data.
2.9 A study of the relationship between rough weight (X) and finished weight (Y) of castings was made. A sample of 12 casings was examined and the data presented below:

| X |  |
| :---: | :---: |
| Rough weight | Y <br> Finished Weight |
| 3.715 | 3.055 |
| 3.685 | 3.020 |
| 3.680 | 3.050 |
| 3.665 | 3.015 |
| 3.660 | 3.010 |
| 3.655 | 3.015 |
| 3.645 | 3.005 |
| 3.630 | 3.010 |
| 3.625 | 2.990 |
| 3.620 | 3.010 |
| 3.610 | 3.005 |
| 3.595 | 2.985 |

a) Make a scatter plot to see the relationship between X and Y .
b) Compute the correlation coefficient, r .

